C4 Exercise 6A (standard integrals)

Note Title 04/07/2007

Here is a list of standard integrals to learn:

① $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$

② $\int e^x \, dx = e^x + C$

③ $\int \frac{1}{x} \, dx = \ln|x| + C$

④ $\int \cos x \, dx = \sin x + C$

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⑤ $\int \sin x \, dx = -\cos x + C$

⑥ $\int \sec^2 x \, dx = \tan x + C$

⑦ $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$

⑧ $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

⑨ $\int \sec x \tan x \, dx = \sec x + C$

1 Integrate the following with respect to x:

a) $3\sec^2 x + \frac{5}{x} + \frac{2}{x^2}$

$$\int 3\sec^2 x \, dx = 3\tan x + C_1$$

$$\int \frac{5}{x} \, dx = 5\ln|x| + C_2$$

$$\int 2x^{-2} \, dx = -2x^{-1} + C_3$$

$$\Rightarrow \int \left(3\sec^2 x + \frac{5}{x} + \frac{2}{x^2}\right) dx = 3\tan x + 5\ln|x| - \frac{2}{x} + C$$

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16 $5e^x - 4\sin x + 2x^3$

$$\int 5e^x \, dx = 5e^x + C_1$$

$$\int -4\sin x \, dx = 4\cos x + C_2$$

$$\int 2x^3 \, dx = \frac{1}{2}x^4 + C_3$$

$$\Rightarrow \int \left(5e^x - 4\sin x + 2x^3\right) dx = 5e^x + 4\cos x + \frac{1}{2}x^4 + C$$

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1c $2(\sin x - \cos x + x)$

$$\int 2\sin x \, dx = -2\cos x + C_1$$

$$\int -2\cos x \, dx = -2\sin x + C_2$$

$$\int 2x \, dx = x^2 + C_3$$

$$\Rightarrow \int 2(\sin x - \cos x + x)\, dx = -2(\sin x + \cos x) + x^2 + C$$

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1d $3\sec x \tan x - \frac{2}{x}$

$$\int 3\sec x \tan x \, dx = 3\sec x + C_1$$

$$\int \frac{-2}{x} \, dx = -2\ln|x| + C_2$$

$$\Rightarrow \int \left(3\sec x \tan x - \frac{2}{x}\right) dx = 3\sec x - 2\ln|x| + C$$

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1e $5e^x + 4\cos x - \frac{2}{x^2}$

$$\int 5e^x \, dx = 5e^x + C_1$$

$$\int 4\cos x \, dx = 4\sin x + C_2$$

$$\int -2x^{-2} \, dx = 2x^{-1} + C_3$$

$$\Rightarrow \int \left(5e^x + 4\cos x - \frac{2}{x^2}\right) dx = 5e^x + 4\sin x + \frac{2}{x} + C$$

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1f $\frac{1}{2x} + 2\operatorname{cosec}^2 x$

$$\int \tfrac{1}{2} \times \tfrac{1}{x}\, dx = \tfrac{1}{2}\ln|x| + C_1$$

$$\int 2\operatorname{cosec}^2 x = -2\cot x + C_2$$

$$\Rightarrow \int \left(\tfrac{1}{2x} + 2\operatorname{cosec}^2 x\right) dx = \tfrac{1}{2}\ln|x| - 2\cot x + C$$

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1g $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$

$$\int \frac{1}{x}\,dx = \ln|x| + C_1$$

$$\int x^{-2}\,dx = -x^{-1} + C_2$$

$$\int x^{-3}\,dx = -\frac{1}{2}x^{-2} + C_3$$

$$\Rightarrow \int \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}\right)dx = \ln|x| - \frac{1}{x} - \frac{1}{2x^2} + C$$

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1h $e^x + \sin x + \cos x$

$$\int e^x \, dx = e^x + C_1$$

$$\int \sin x \, dx = -\cos x + C_2$$

$$\int \cos x \, dx = \sin x + C_3$$

$$\Rightarrow \int \left(e^x + \sin x + \cos x\right) dx = e^x - \cos x + \sin x + C$$

1i $2\operatorname{cosec} x \cot x - \sec^2 x$

$$\int 2\operatorname{cosec} x \cot x \, dx = -2\operatorname{cosec} x + C_1$$

$$\int -\sec^2 x = -\tan x + C_2$$

$$\Rightarrow \int (2\operatorname{cosec} x \cot x - \sec^2 x) dx = -2\operatorname{cosec} x - \tan x + C$$

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1j $e^x + \frac{1}{x} - \operatorname{cosec}^2 x$

$$\int e^x \, dx = e^x + C_1$$

$$\int \frac{1}{x} \, dx = \ln|x| + C_2$$

$$\int -\operatorname{cosec}^2 x \, dx = \cot x + C_3$$

$$\Rightarrow \int \left(e^x + \frac{1}{x} - \operatorname{cosec}^2 x\right) dx = e^x + \ln|x| + \cot x + C$$

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2a Find the following integrals:

$$\int \left(\frac{1}{\cos^2 x} + \frac{1}{x^2} \right) dx = \int \left(\sec^2 x + x^{-2} \right) dx$$

$$= \tan x - \frac{1}{x} + C$$

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26 $\int \left(\frac{\sin x}{\cos^2 x} + 2e^x \right) dx \equiv \int \left(\sec x \tan x + 2e^x \right) dx$

$= \sec x + 2e^x + C$

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1c $\int \left(\frac{1 + \cos x}{\sin^2 x} + \frac{1 + x}{x^2} \right) dx \equiv \int \left(\operatorname{cosec}^2 x + \operatorname{cosec} x \cot x + x^{-2} + \frac{1}{x} \right) dx$

$= -\cot x - \operatorname{cosec} x - \frac{1}{x} + \ln|x| + C$

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2d $\int \left(\frac{1}{\sin^2 x} + \frac{1}{x} \right) dx = \int \left(\operatorname{cosec}^2 x + \frac{1}{x} \right) dx$

$= -\cot x + \ln|x| + C$

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2e $I \equiv \int \sin x \left(1 + \sec^2 x\right) dx = \int \left(\sin x + \frac{\sin x}{\cos x} \times \frac{1}{\cos x}\right) dx$

$= \int \left(\sin x + \sec x \tan x\right) dx$

$I = -\cos x + \sec x + C$

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2f $I = \int \left(\cos x \left(1 + \operatorname{cosec}^2 x \right) \right) dx = \int \left(\cos x + \frac{\cos x}{\sin x} \times \frac{1}{\sin x} \right) dx$

$= \int \left(\cos x + \operatorname{cosec} x \cot x \right) dx$

$I = \sin x - \operatorname{cosec} x + C$

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2g $I = \int \operatorname{cosec}^2 x \left(1 + \tan^2 x\right) dx = \int \left(\operatorname{cosec}^2 x + \frac{1}{\sin^2 x} \times \frac{\sin^2 x}{\cos^2 x}\right) dx$

$= \int \left(\operatorname{cosec}^2 x + \sec^2 x\right) dx$

$I = -\cot x + \tan x + C$

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2h $\int \sec^2 x \left(1 - \cot^2 x\right) dx = \int \left(\sec^2 x - \frac{1}{\cancel{\cos^2 x}} \times \frac{\cancel{\cos^2 x}}{\sin^2 x}\right) dx$

$= \int \left(\sec^2 x - \operatorname{cosec}^2 x\right) dx$

$= \tan x + \cot x + C$

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2i $\int \sec^2 x \left(1 + e^x \cos^2 x\right) dx \equiv \int \left(\sec^2 x + e^x\right) dx$

$= \tan x + e^x + C$

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2j $$\int \left(\frac{1 + \sin x}{\cos^2 x} + \cos^2 x \sec x \right) dx$$

$$\equiv \int \left(\sec^2 x + \sec x \tan x + \cos x \right) dx$$

$$= \tan x + \sec x + \sin x + C$$